

# Section 10.3 — Hypothesis Testing for Population Means ( $\sigma$ unknown)

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Performing a Hypothesis Test

Hypothesis test for Population Means ( $\sigma$  unknown)

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# Basic Steps

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3. Gather data and calculate the necessary sample statistics.
4. Draw a conclusion and interpret the decision.

# Types of Hypothesis Tests

Symbol in $H_a$	Tail
$\neq$	Two-tailed test
$<$	Left-tailed test
$>$	Right-tailed test

# Critical values

## Definition (Critical region)

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The **critical values** are the values that separate the critical region from the values of the test statistic that do not lead to rejection of the null hypothesis. These are determined by  $\alpha$ .

# $p$ -values

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## Determining $p$ -values

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Left-tailed	$p$ -value = area to the <i>left</i> of the test statistic
Right-tailed	$p$ -value = area to the <i>right</i> of the test statistic
Two-tailed	$p$ -value = <i>twice</i> the area in the tail beyond the test statistic

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# Make a decision

## *p*-value method

If  $p\text{-value} \leq \alpha$ , reject  $H_0$ . Otherwise, fail to reject  $H_0$ .

## Critical value method

If the test statistic is in the critical region, reject  $H_0$ . Otherwise, fail to reject.

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# Test Statistic

with

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$df = n - 1$$

## Teachers' salaries

Teacher's salaries in one district are very low, so low that educators in that state regularly complain about their compensation. The state mean is \$33,600, but teachers in one district claim that the mean in their district significantly lower. They survey a simple random sample of 22 teachers in the district and calculate the mean salary of \$32,400 with a standard deviation of \$1520. Test the teacher's claim at the 0.01 level of significance.

# Children's Clothing

A children's clothing company sells hand-smocked dresses for girls. The length of one particular size dress is designed to be 26 inches. The company regularly tests the lengths of the garments to ensure quality control. If the mean length is found to be significantly longer or shorter than 26 inches, then the machines must be adjusted. The most recent simple random sample of 28 dresses had a mean length of 26.30 inches with a standard deviation of 0.77 inches. Perform a hypothesis test on the accuracy of the machines at the 0.01 level of significance.