Section 4.1 — Intro to Probability

Chris Godbout

Outline

Definitions and Notation

Definitions

Definition (Probability Experiment)

A probability experiment (or *trial*) is any process with a result determined by chance.

Definitions

Definition (Probability Experiment)

A probability experiment (or *trial*) is any process with a result determined by chance.

Definition (Outcomes)

An outcome is an individual result that is possible for a trial.

Definitions

Definition (Probability Experiment)

A probability experiment (or *trial*) is any process with a result determined by chance.

Definition (Outcomes)

An outcome is an individual result that is possible for a trial.

Definition (Sample Space)

The sample space is the set of all possible outcomes for a given probability experiment.

Definitions

Definition (Probability Experiment)

A probability experiment (or *trial*) is any process with a result determined by chance.

Definition (Outcomes)

An outcome is an individual result that is possible for a trial.

Definition (Sample Space)

The sample space is the set of all possible outcomes for a given probability experiment.

Definition (Event)

An event is a collection of outcomes from the sample space.

Example

If there are 3 births and we're interested only in whether the children are boys or girls, what is the sample space? What are some other events?

Notation

P denotes probability

Notation

P	denotes probability
A,B,C, etc.	denote specific events

Notation

P	denotes probability
A,B,C, etc.	denote specific events
P(A)	is the probability that *A* occurs.

Basics of Probability

Different Approaches

How do we compute $P(A)$?

Relative Frequency Approximation of Probability

Conduct (or observe) a procedure and count the number of times that event A occurs. $P(A)$ is *approximated* by

$$P(A) = \frac{\text{number of times } A \text{ occurred}}{\text{number of times procedure was repeated}}$$

Classical Approach

Assume that procedure has multiple possible outcomes and *each outcome is equally likely*. Then

$$P(A) = \frac{\text{number of ways } A \text{ occur}}{\text{number of outcomes}}$$

Law of Large Numbers

Theorem (Law of Large Numbers)

As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.

Law of Large Numbers

Theorem (Law of Large Numbers)

As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.

Theorem (Gambler's Fallacy)

The mistaken belief that if something happens more frequently during some period then it will happen less frequently in the future.

Babies!!!!

If there are two children, what's the probability of 2 boys, 2 girls, or a boy and a girl?

Civil Rights Act of 1964

Table 1: Civil Rights Act of 1964 Votes

	Yes	No
Democrats	152	96
Republicans	138	34

What's the probability that a randomly selected congressman or senator voter *for* the bill?

Thanksgiving

If a year is selected at random, what is the probability that Thanksgiving falls on a Wednesday? What about on a Thursday?

Thanksgiving

If a year is selected at random, what is the probability that Thanksgiving falls on a Wednesday? What about on a Thursday?

- P(Thanksgiving on Wednesday) = 0

Thanksgiving

If a year is selected at random, what is the probability that Thanksgiving falls on a Wednesday? What about on a Thursday?

- P(Thanksgiving on Wednesday) = 0
- P(Thanksgiving on Thursday) = 1

Possible probabilities

What are the largest and smallest possible probabilities?

Likelihood of Events

Definitions

- An event is unlikely if its probability is very small (perhaps less than 0.05).
- An event is likely if its probability is very large.
- An event is impossible if its probability is 0.
- An event is certain if its probability is 1.