

Section 5.1 — Discrete Random Variables

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Outline

Introduction

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MOAR EXAMPLES!!!

Introduction

Definition (Random Variable)

A **random variable** is a variable (typically X , Y , or Z) that has a single numerical value, determined by chance, for each outcome of a procedure.

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Definition (Probability Distribution)

A **probability distribution** is a description that gives the probability for each value of the random variable. It is often expressed in the format of a table, formula, or graph.

Definition (Discrete Random Variable)

A **discrete random variable** has a collection of values that is finite or determined by a counting process.

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3. $\sum P(X = x) = 1$

Examples

Table 1: Responses to the question "Should marijuana use be legal?"

| Response | $P(X = x)$ |
|------------|------------|
| Yes | 0.409 |
| No | 0.520 |
| Don't Know | 0.070 |

Is this a probability distribution?

Table 2: Responses to the question "On which interview should a candidate begin salary negotiations?"

| Number of Interviews x | $P(X = x)$ |
|--------------------------|------------|
| 1 | 0.30 |
| 2 | 0.26 |
| 3 | 0.10 |

Is this a probability distribution?

$$P(X = x) = \frac{x}{10} \text{ for } x = 0, 1, 2, 3, 4$$

Is this a probability distribution?

Parameters

Expected Value

The **expected value** for a discrete random variable X is equal to the mean of the probability distribution. It is given by

$$E(X) = \mu = \sum (x_i \cdot P(X = x_i))$$

Definition (Variance)

The variance of a probability distribution is either

$$\sigma^2 = \sum \left((x_i - \mu)^2 \cdot P(X = x_i) \right)$$

or

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Definition (Standard Deviation)

The standard deviation of a probability distribution is

$$\sigma = \sqrt{\sum (x_i^2 \cdot P(X = x_i)) - \mu^2}$$

MOAR EXAMPLES!!!

Genetic Disorders

Four males with an X-linked genetic disorder have one child each. The random variable x is the number of children among the four who inherit the genetic disorder.

Table 3: Number of children among with disorder

| x | $P(X = x)$ |
|-----|------------|
| 0 | 0.0625 |
| 1 | 0.2500 |
| 2 | 0.3750 |
| 3 | 0.2500 |
| 4 | 0.0625 |

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- What is the probability of winning?
- What is the expected value of your winnings?