Section 5.2 - Binomial Probability Distributions

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Outline

Introduction

Definition

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2. The trials must be *independent*.
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4. The probability must remain the same for all trials.
5. The random variable X counts the number of successes in n trials.

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- $P(X = x)$ is the probability of getting exactly x successes.
- $P(X < x)$ is the probability of getting *fewer* than x successes.

Formula

Theorem (Binomial Probability Formula)

$$P(X = x) = {}_{n}C_{x} \cdot p^{x} \cdot (1 - p)^{n-x}$$

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Standard Deviation

The standard deviation of a binomial probability distribution is

$$\sigma = \sqrt{np(1-p)}$$

Examples

Multiple-choice tests

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- What is p?
- What is $P(X = 7)$?
- What is $E(X)$?
- What is the probability that the student gets at least 7 correct?

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Assume that 45% of all people have blood that is in Group O.

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- What is the probability that exactly 3 out of 8 people have blood in Group O?
- What is the probability that no more than 3 out of 8 people have blood in Group O?
- What is the probability that exactly 16 out of 21 people have blood in Group O?