

Section 4.1 — Intro to Probability

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Outline

Definitions and Notation

Basics of Probability

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The **sample space** is the set of all possible outcomes for a given probability experiment.

Definition (Event)

An **event** is a collection of outcomes from the sample space.

Example

If there are 3 births and we're interested only in whether the children are boys or girls, what is the sample space? What are some other events?

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Notation

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$P(A)$ is the probability that A occurs.

Basics of Probability

Different Approaches

How do we compute $P(A)$?

Relative Frequency Approximation of Probability

Conduct (or observe) a procedure and count the number of times that event A occurs. $P(A)$ is *approximated* by

$$P(A) = \frac{\text{number of times A occurred}}{\text{number of times procedure was repeated}}$$

Classical Approach

Assume that procedure has multiple possible simple events and *each simple event is equally likely*. Then

$$P(A) = \frac{\text{number of ways } A \text{ occur}}{\text{number of different simple events}}$$

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Theorem (Gambler's Fallacy)

The mistaken belief that if something happens more frequently during some period then it will happen less frequently in the future.

Babies!!!!

If there are two children, what's the probability of 2 boys, 2 girls, or a boy and a girl?

Civil Rights Act of 1964

Table 1: Civil Rights Act of 1964 Votes

	Yes	No
Democrats	152	96
Republicans	138	34

What's the probability that a randomly selected congressman or senator voter *for* the bill?

Thanksgiving

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- $P(\text{Thanksgiving on Thursday}) = 1$

Possible probabilities

What are the largest and smallest possible probabilities?

Complementary Events

Definition

The **complement** of event A , denoted \bar{A} , consists of all possible outcomes in which A does *not* occur.

If A is the event that a random congressman voted yes for the CRA'64, what is \bar{A} ? What is $P(\bar{A})$?

Likelihood of Events

Definitions

- An event is **unlikely** if its probability is very small (perhaps less than 0.05).
- An event is **likely** if its probability is very large.
- An event is **impossible** if its probability is 0.
- An event is **certain** if its probability is 1.
- An event is **unusual** if it has more possible outcomes than we typically expect.